

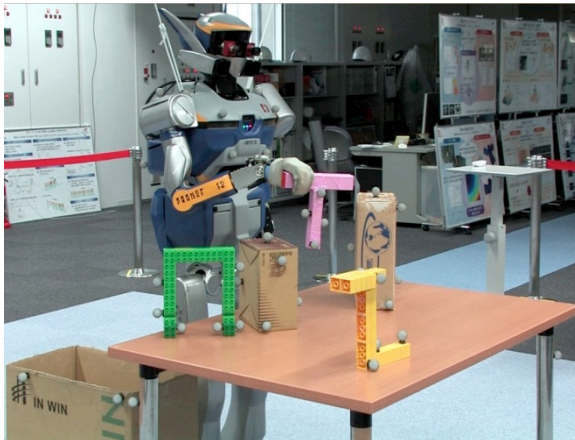


# Introduction to State-of-the-art Motion Planning Algorithms

*Presented by*  
Konstantinos Tsianos



# Robots need to move!



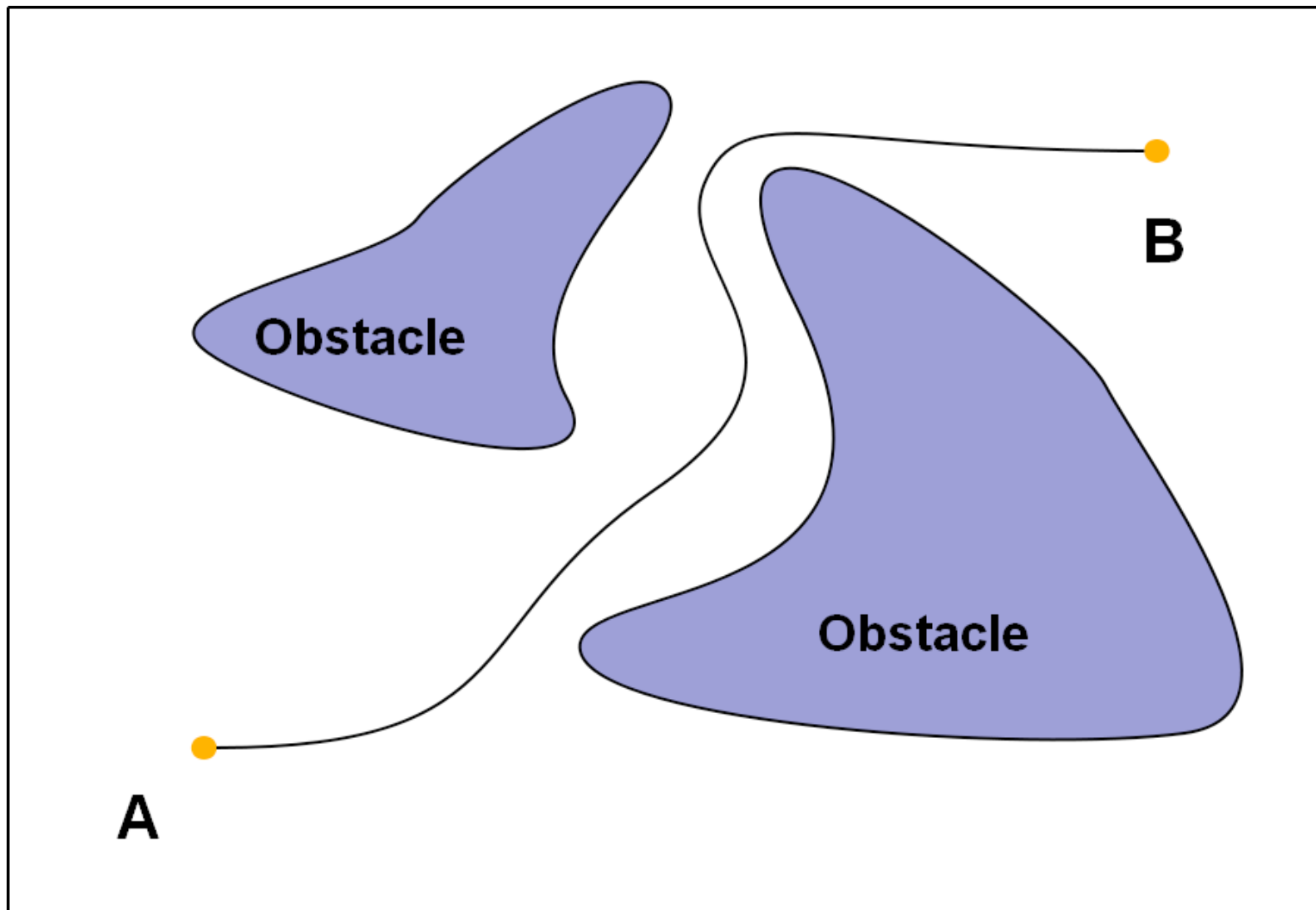


# Motion

- Robot motion must be continuous
- Geometric constraints
- Dynamic constraints
- Safety constraints
- Execution constraints



# Path planning – *A to B* paradigm



A real robot has a shape





# A robot as a point in its **Configuration Space**

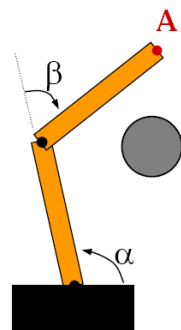
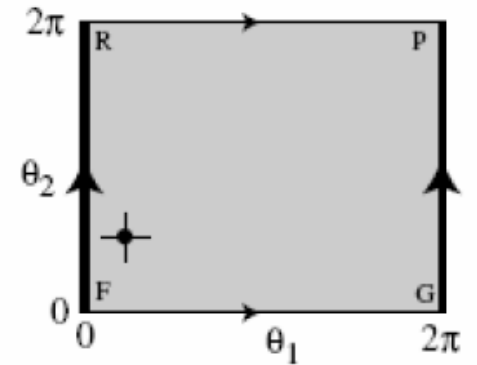
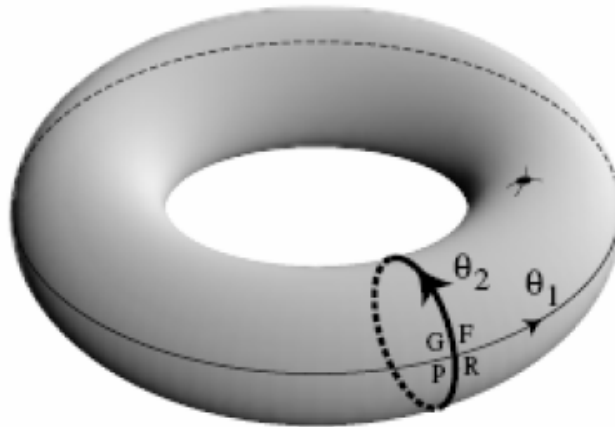
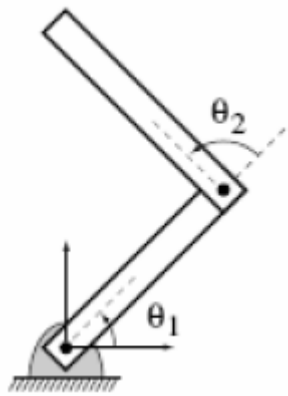
- Given its geometry, a robot is fully specified by a minimal set of parameters  $p_1, p_2, \mathbf{K}, p_n$ 
  - *Generalized coordinates*
  - *Degrees of freedom*
  - *Configurations space parameters*
- If  $p_i \in X_i$ , the configuration space (*C-Space*) is a manifold

$$Q = X_1 \times X_2 \times \mathbf{K} \times X_n$$

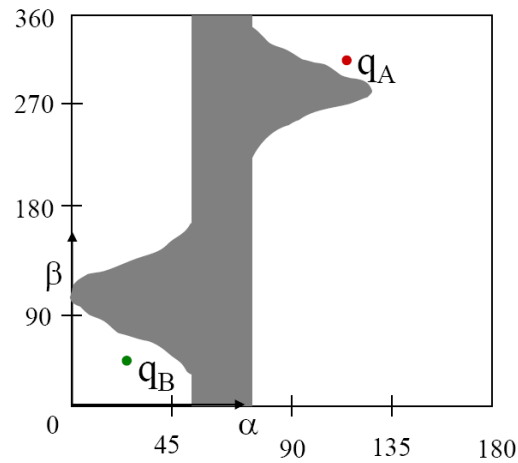




# Configuration space of a 2-link arm

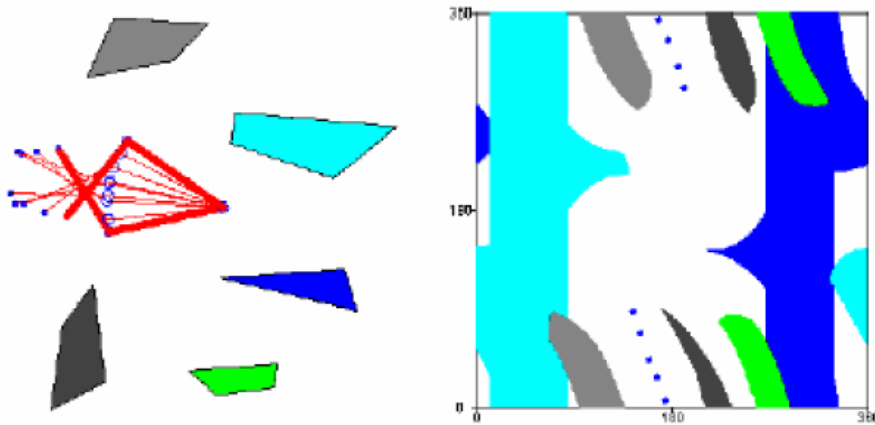


B



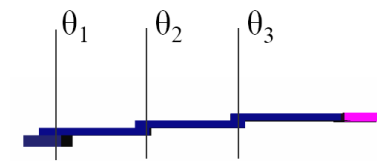


# Some more examples



2-link planar arm  
among 5 obstacles

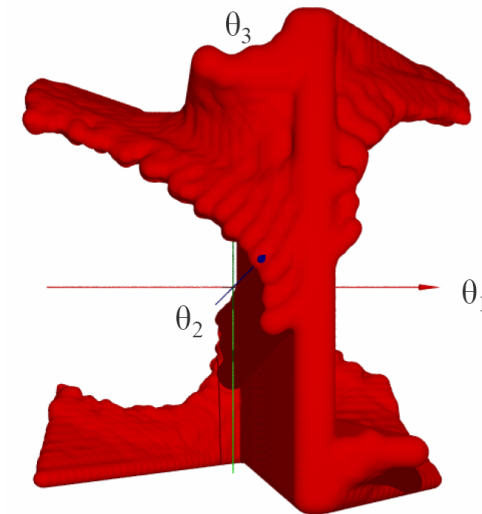
3-link  
planar arm



TOP  
VIEW



workspace



C-space





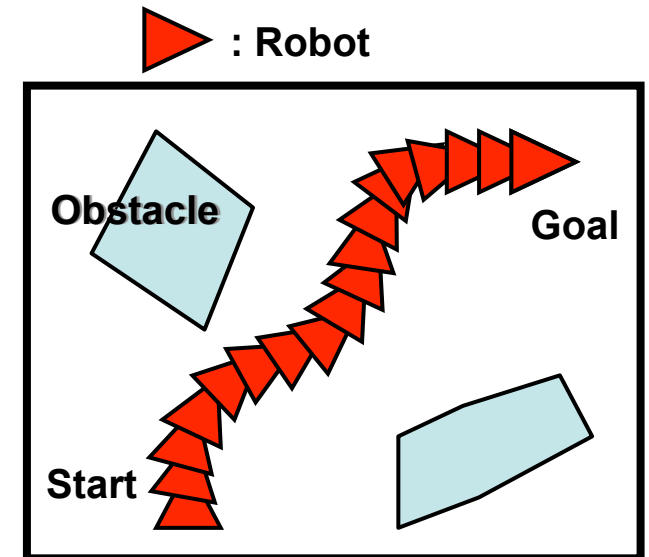
# Path Planning

## Given:

- World geometry
- Robot's geometry
- Start and goal configuration

## Compute:

A collision-free, feasible path to the goal



Workspace  
Model

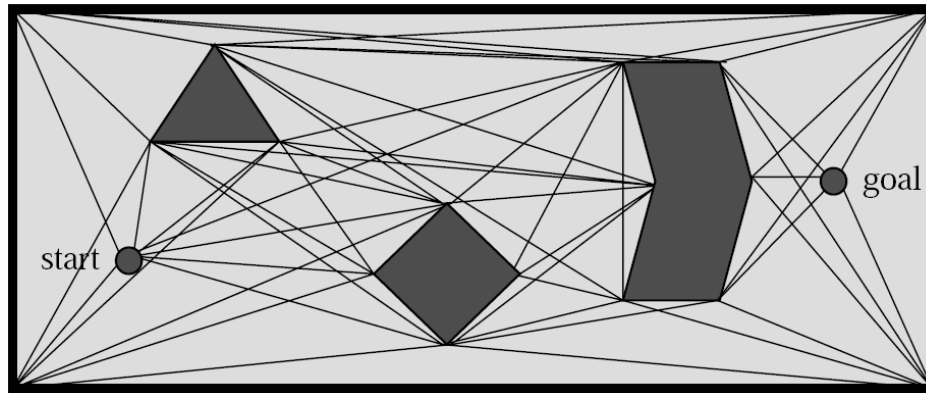
Robot Model

**Motion  
Planner**

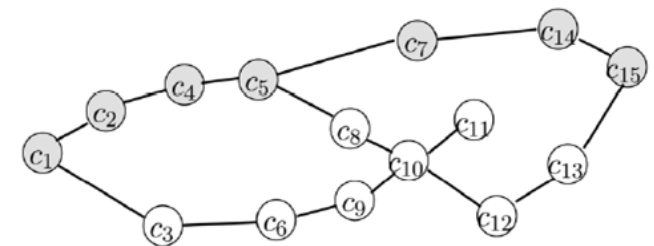
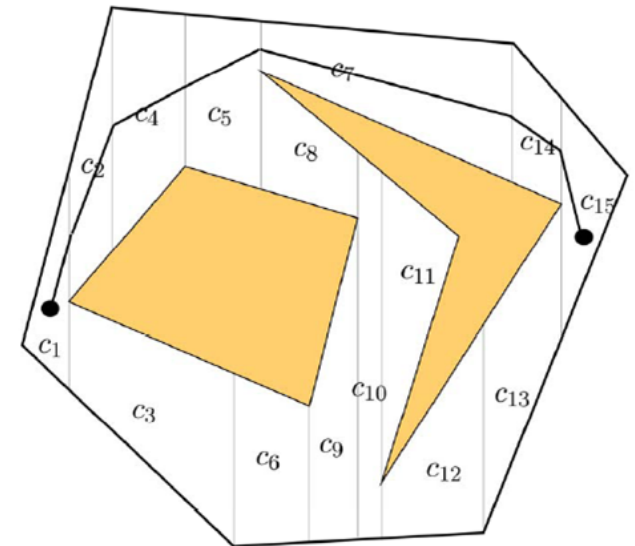
Collision-free  
Path



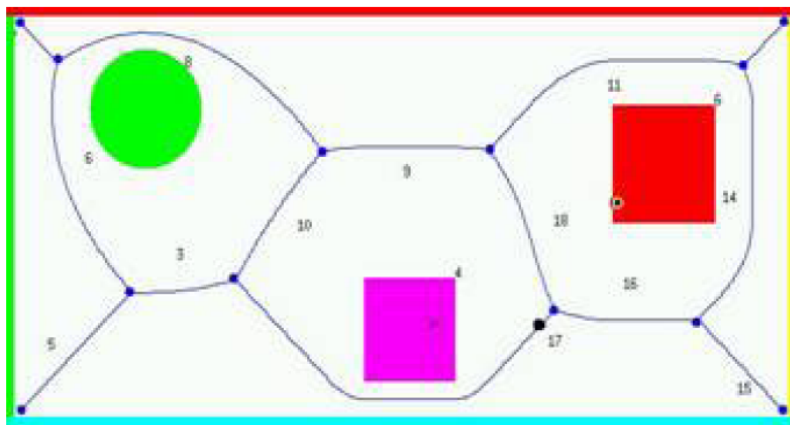
# Complete planners - Examples



Visibility roadmap



Trapezoidal decomposition



Voronoi diagram



# Path planning is hard..

## PROBLEM

## COMPLEXITY

|                      |
|----------------------|
| Sofa Mover (3DOF)    |
| Piano Mover (6DOF)   |
| n Disks in the Plane |
| n Link Chain in 3D   |
| Generalized Mover    |

|  |
|--|
| $O(n^{2+\epsilon})$ - not implemented [HS96] |
| Polynomial – no practical algorithm [SS83]   |
| NP-Hard [SS83]                               |
| PSPACE-Complete [HSS87]                      |
| PSPACE-Complete [Canny88]                    |

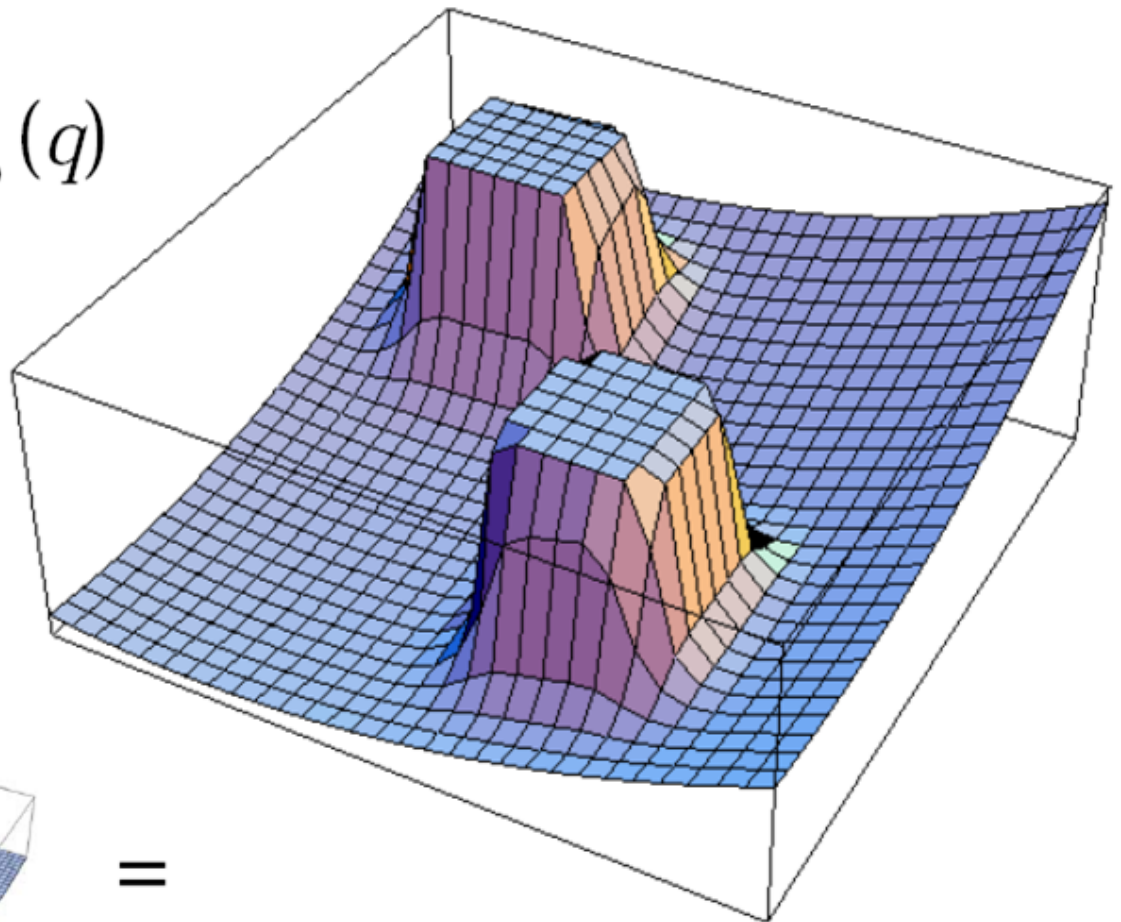
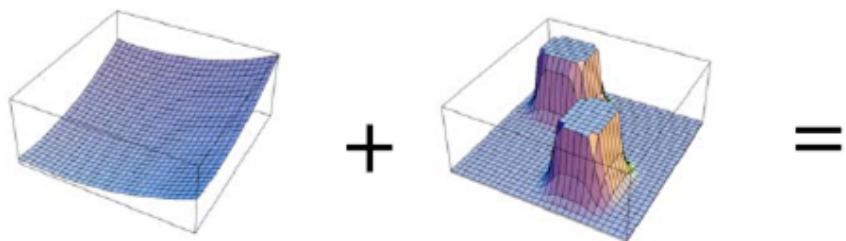
- Understanding the structure of C-spaces is a very hard mathematical challenge
- C-spaces for most interesting problems are high dimensional



# Potential field planners

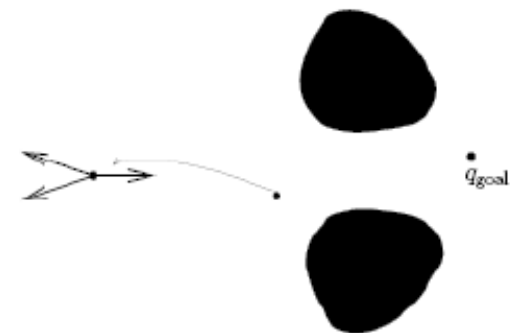
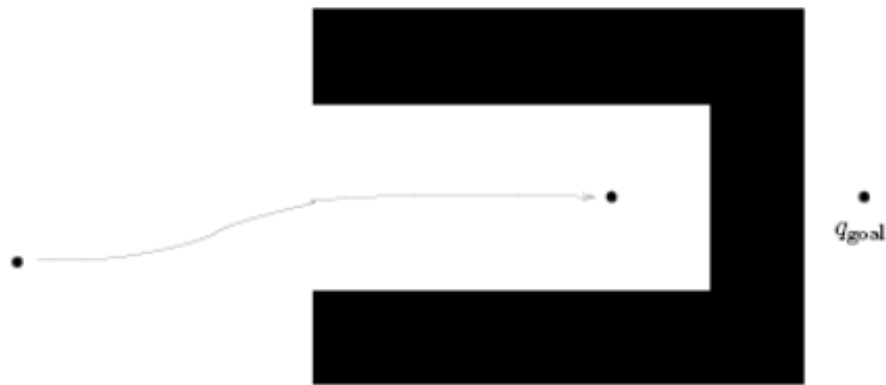
$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

$$F(q) = -\nabla U(q)$$





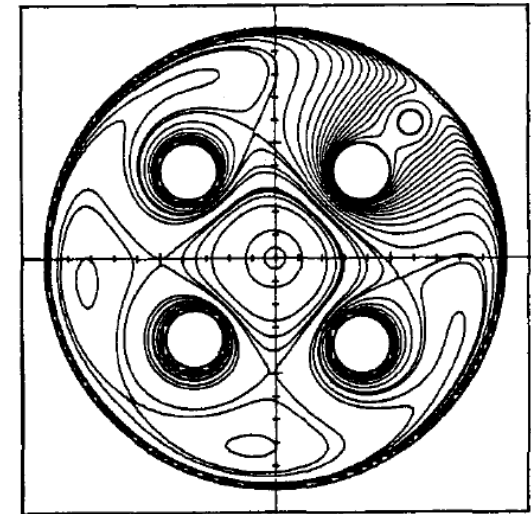
# Local minima problem





# Navigation functions

- Exact navigation functions mathematically challenging general (RK91)



are

- Approximate navigation functions are intractable in many dimensions

|   |    |    |    |    |    |    |    |    |    |   |    |    |    |    |    |    |
|---|----|----|----|----|----|----|----|----|----|---|----|----|----|----|----|----|
| 7 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 9  | 9  | 9  | 9  | 9  | 9  |
| 6 | 17 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8  | 8  | 8  | 8  | 8  | 8  |
| 5 | 17 | 16 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8  | 7  | 7  | 7  | 7  | 7  |
| 4 | 17 | 16 | 15 | 15 | 1  | 1  | 1  | 1  | 1  | 1 | 1  | 1  | 6  | 6  | 6  | 6  |
| 3 | 17 | 16 | 15 | 14 | 1  | 1  | 1  | 1  | 1  | 1 | 1  | 1  | 5  | 5  | 5  | 5  |
| 2 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9  | 8 | 7  | 6  | 5  | 4  | 4  | 4  |
| 1 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9  | 8 | 7  | 6  | 5  | 4  | 3  | 3  |
| 0 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9  | 8 | 7  | 6  | 5  | 4  | 3  | 2  |
|   | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9 | 10 | 11 | 12 | 13 | 14 | 15 |





Planning does not require understanding the structure of the configuration space!

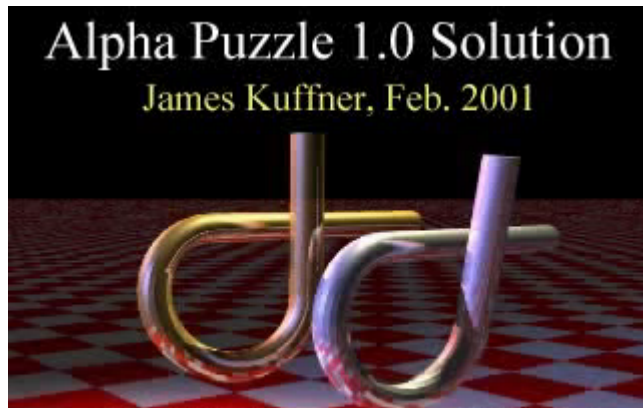


# Sampling-based Planners

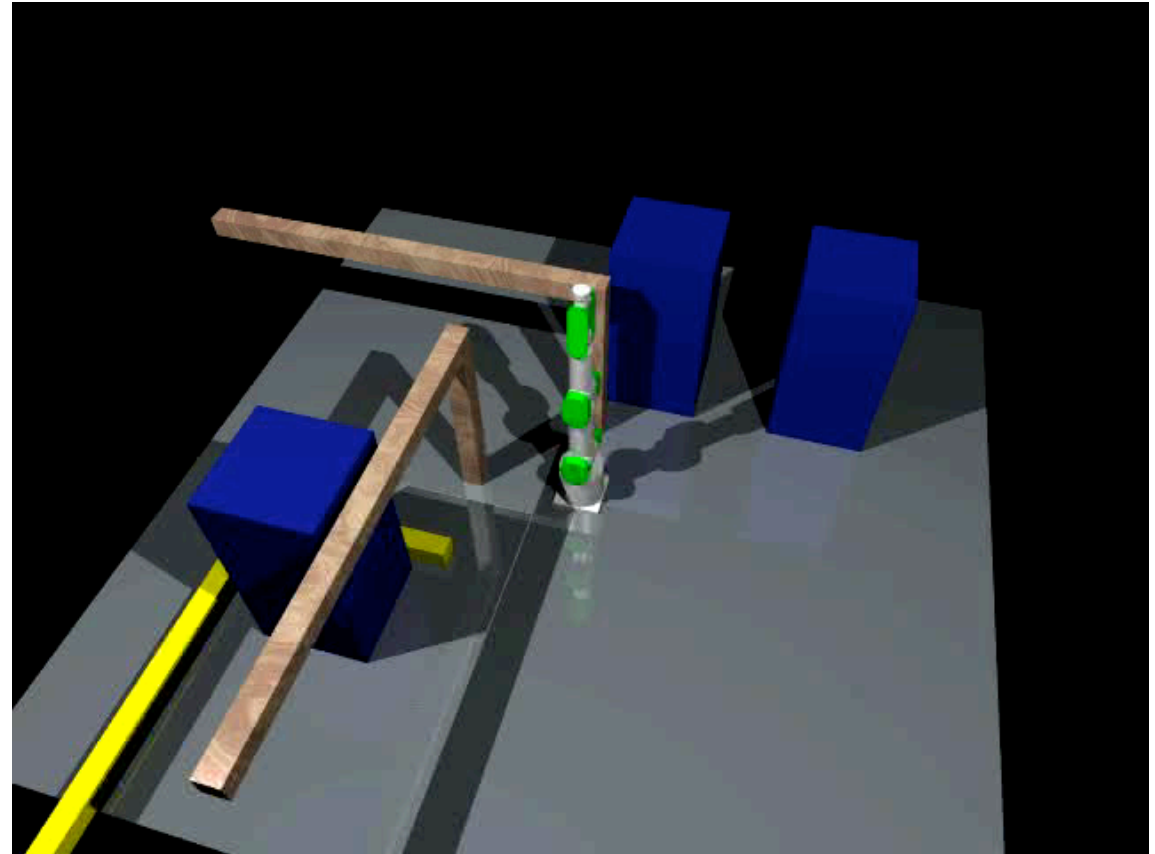
- Try to capture the connectivity of the C-Space without explicitly constructing it
- Use sampling – possible due to development of fast collision checking algorithms
- Result in very efficient algorithms that showed solutions to many previously intractable geometric planning problems



# Two difficult examples

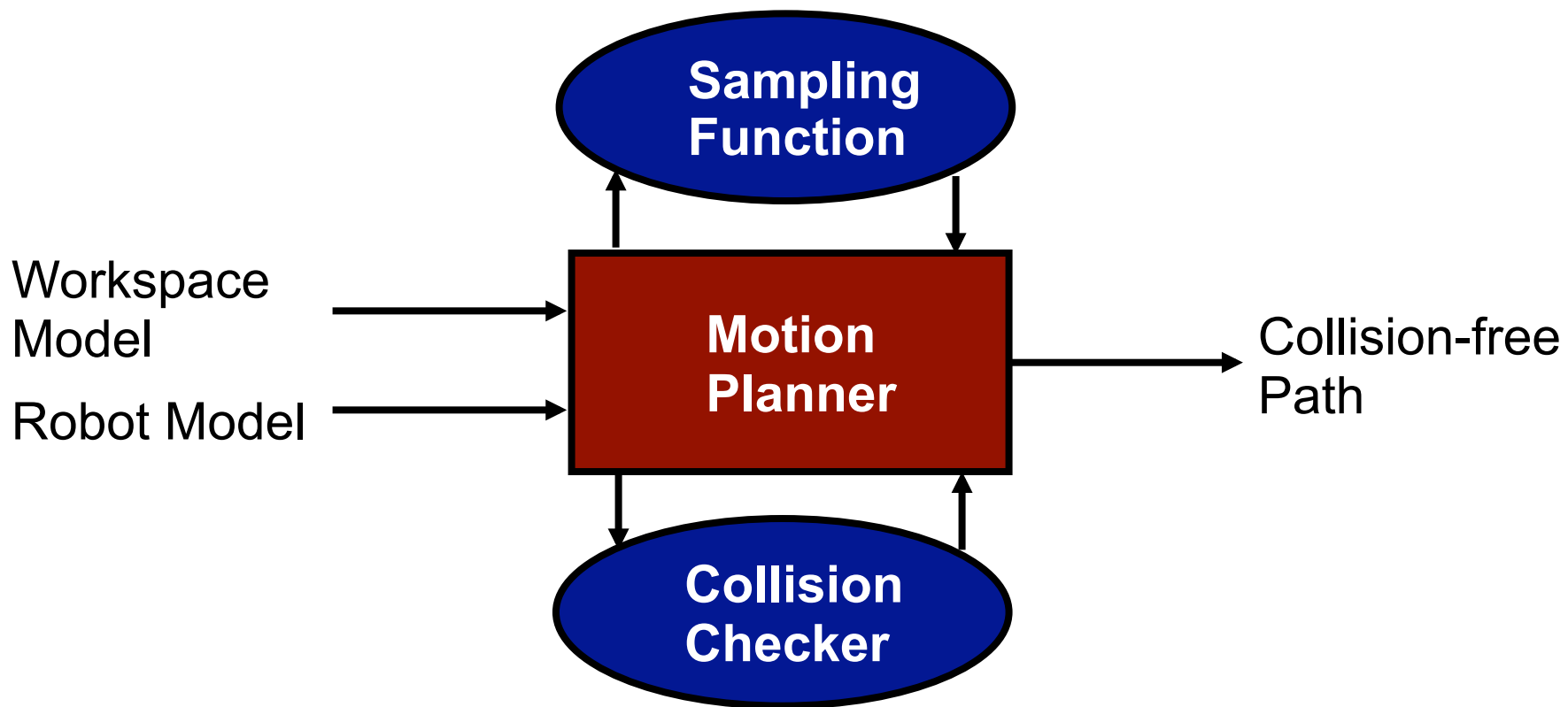


model by DSMFT group, Texas A&M Univ.  
original model by Boris Yamrom, GE



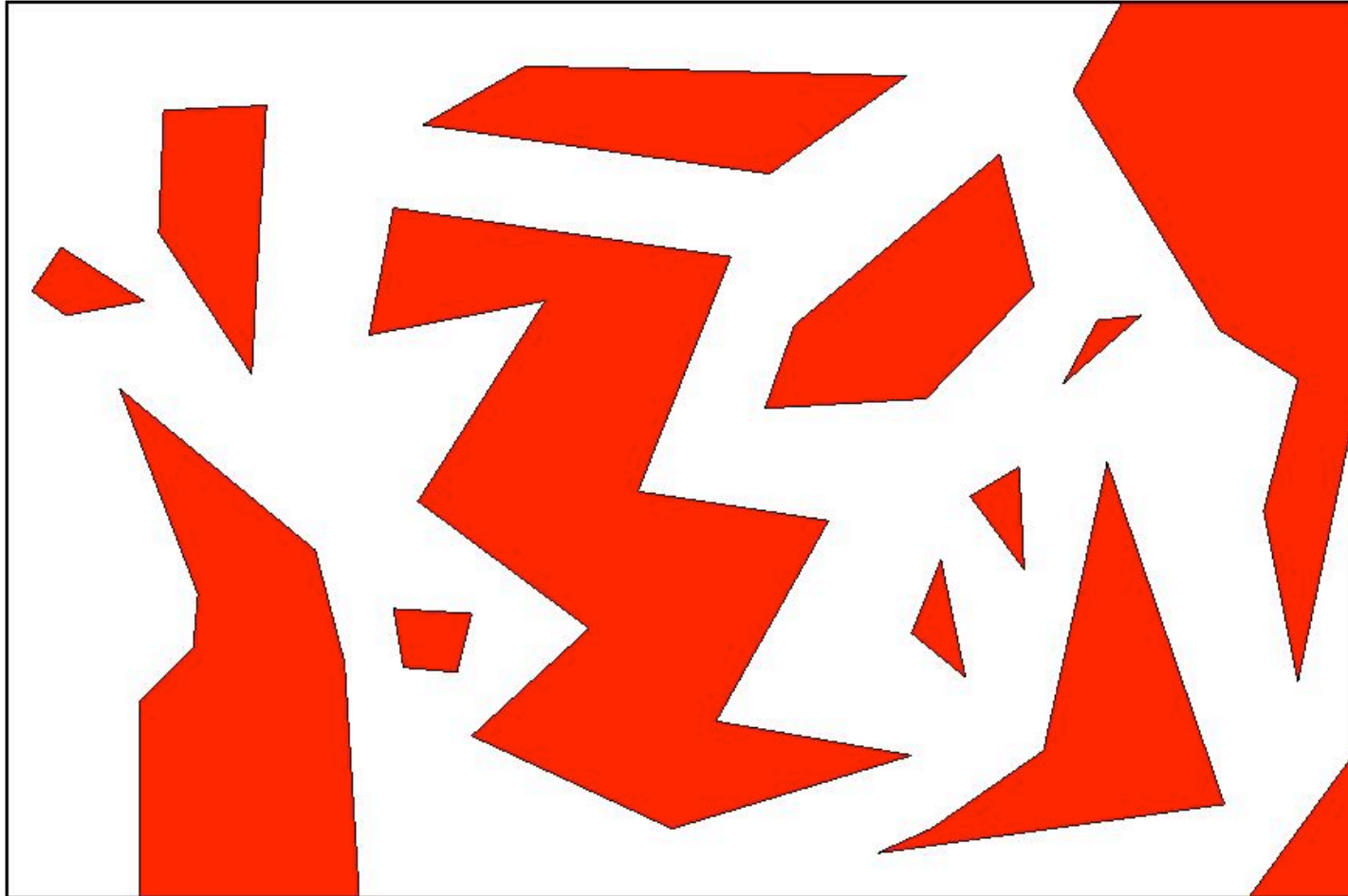


# Sampling-based planning



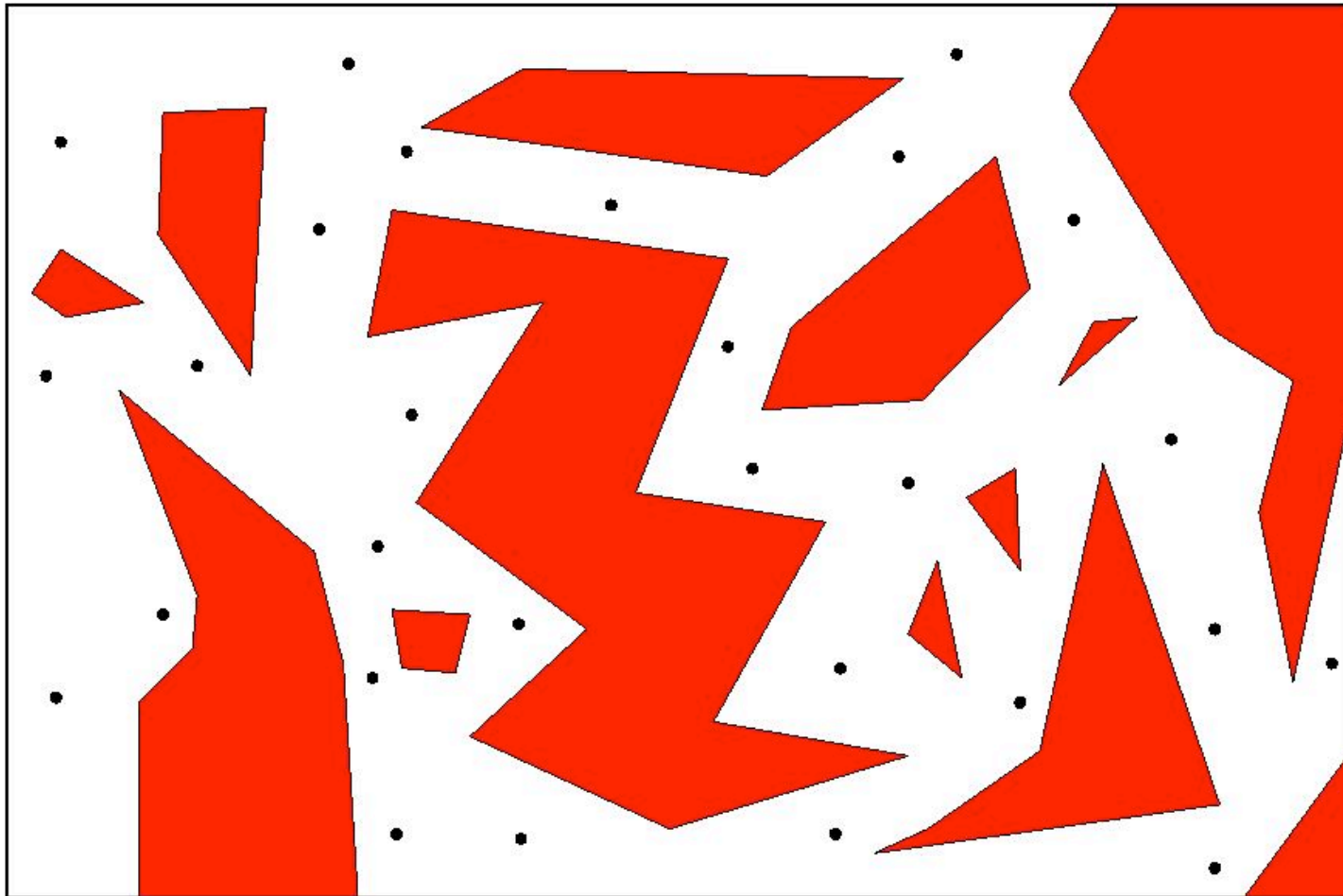


# Probabilistic Roadmap (PRM)





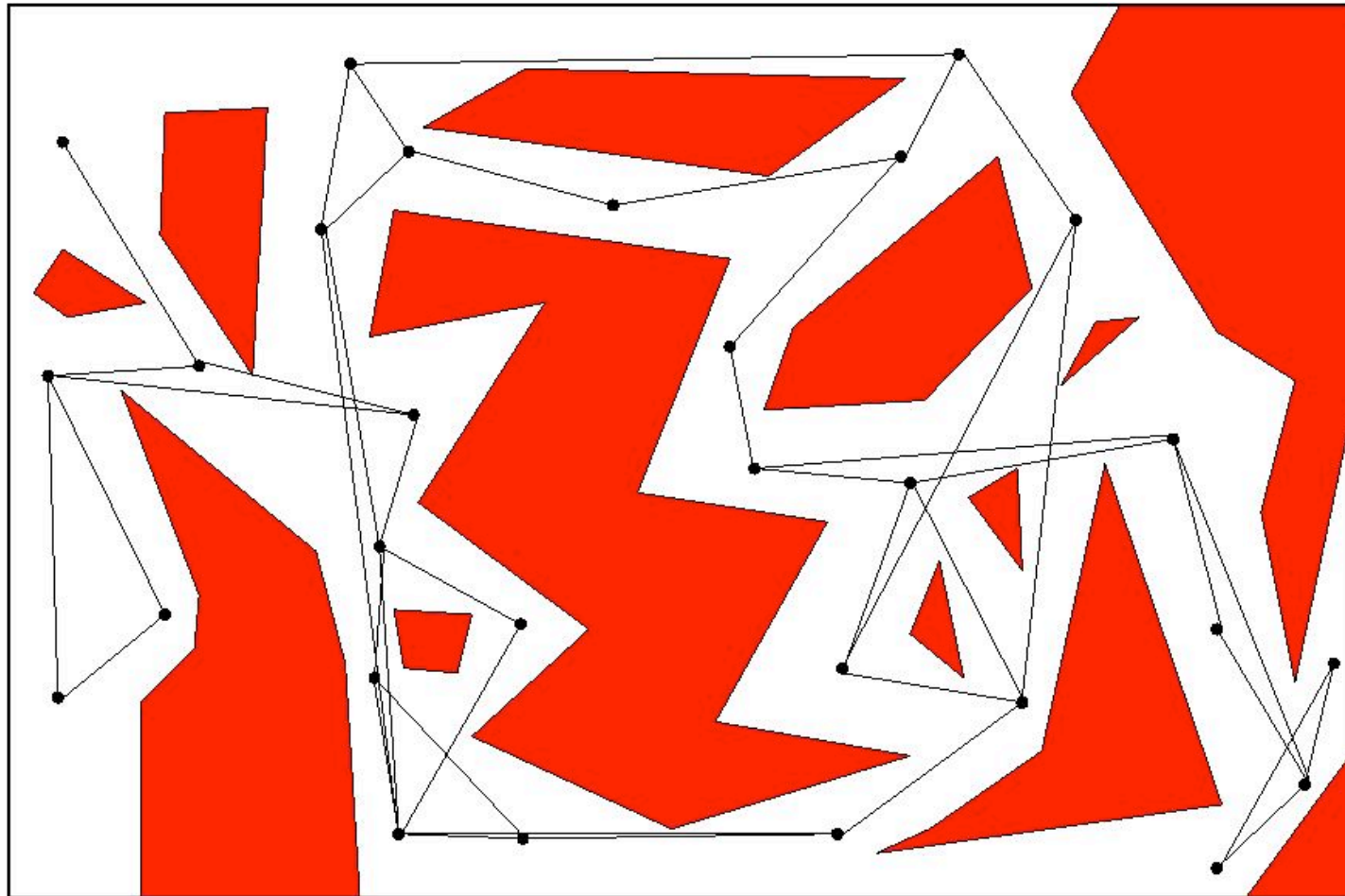
# Probabilistic Roadmap (PRM)





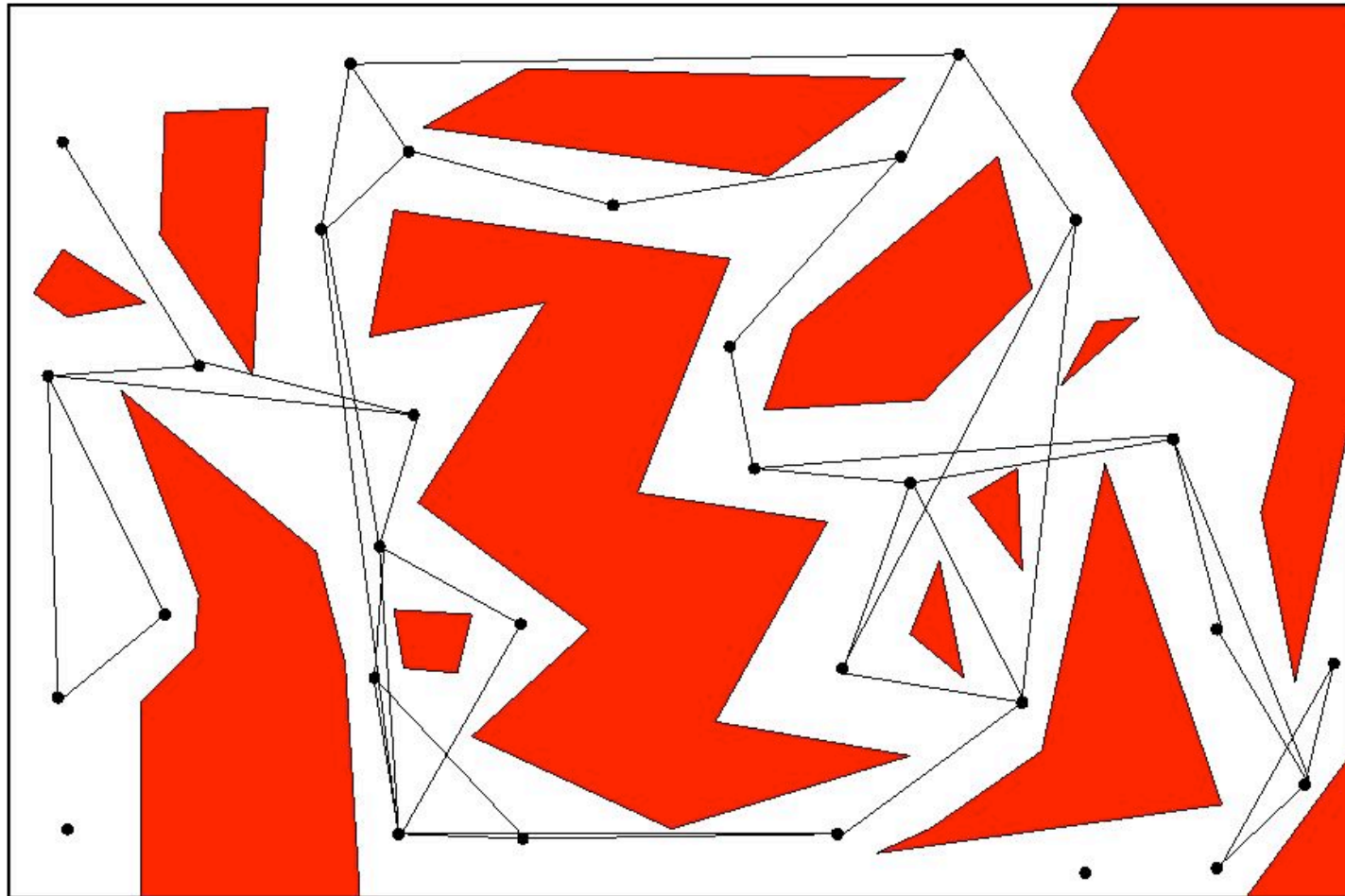


# Probabilistic Roadmap (PRM)



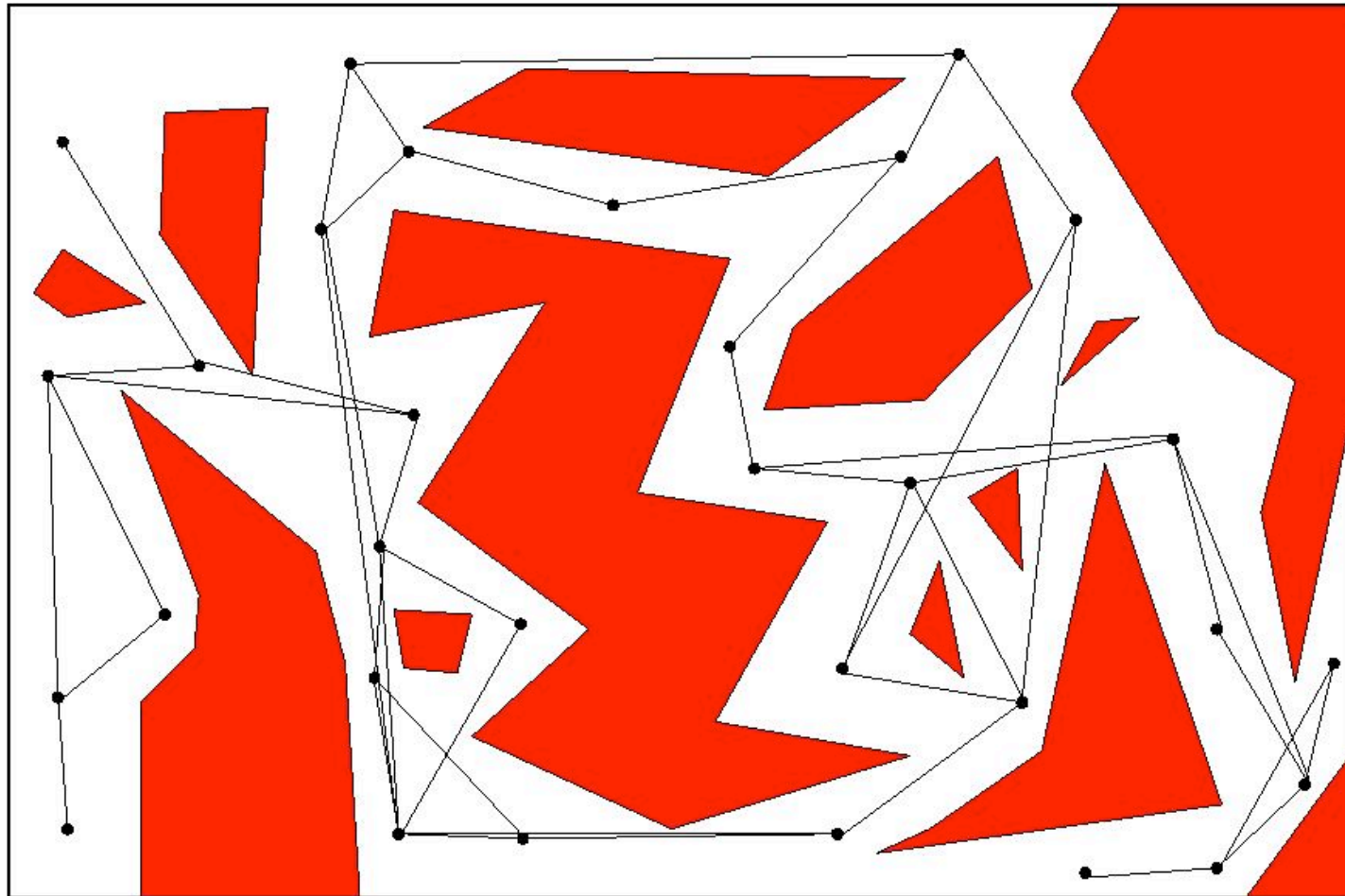


# Probabilistic Roadmap (PRM)



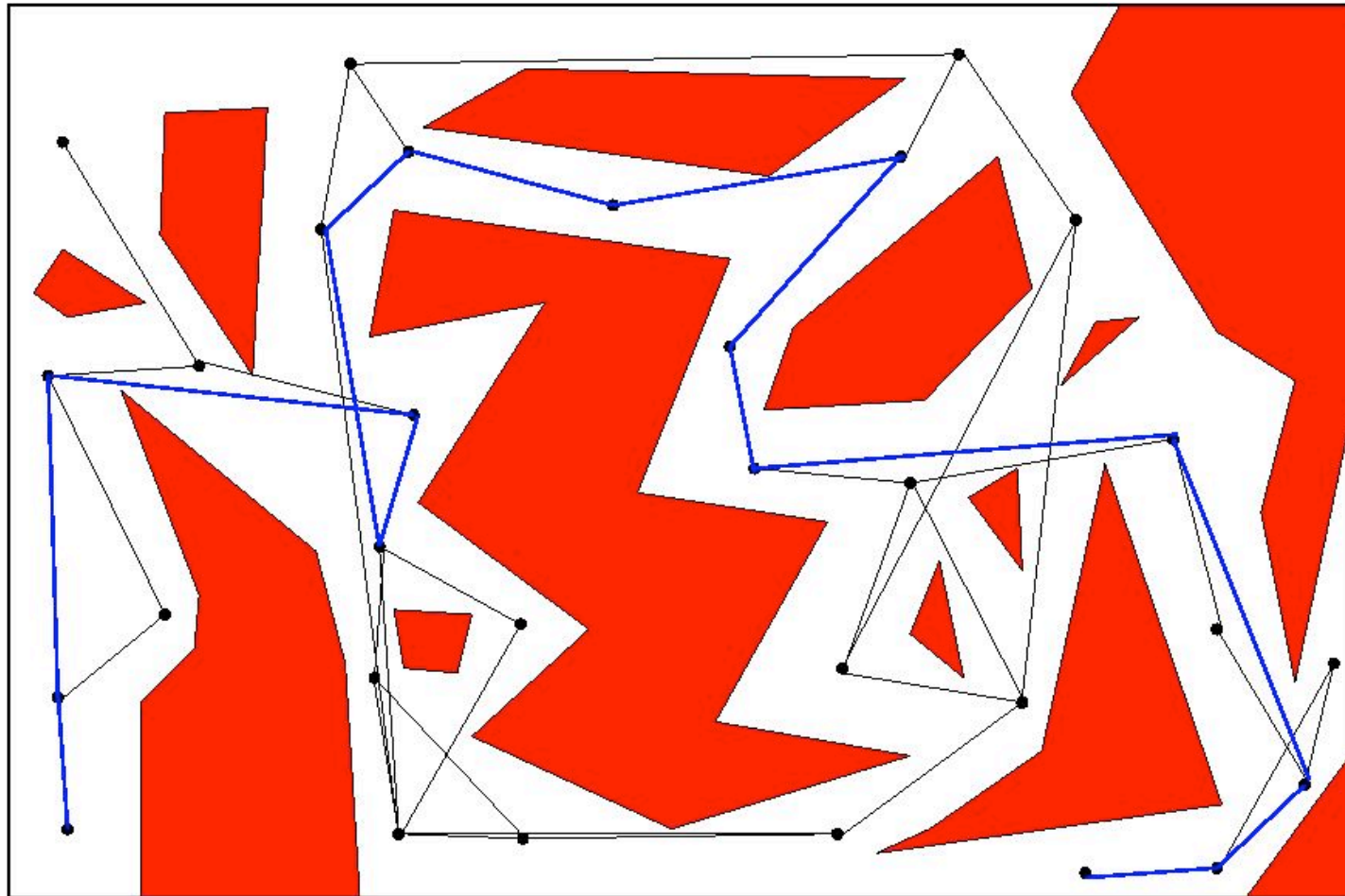


# Probabilistic Roadmap (PRM)





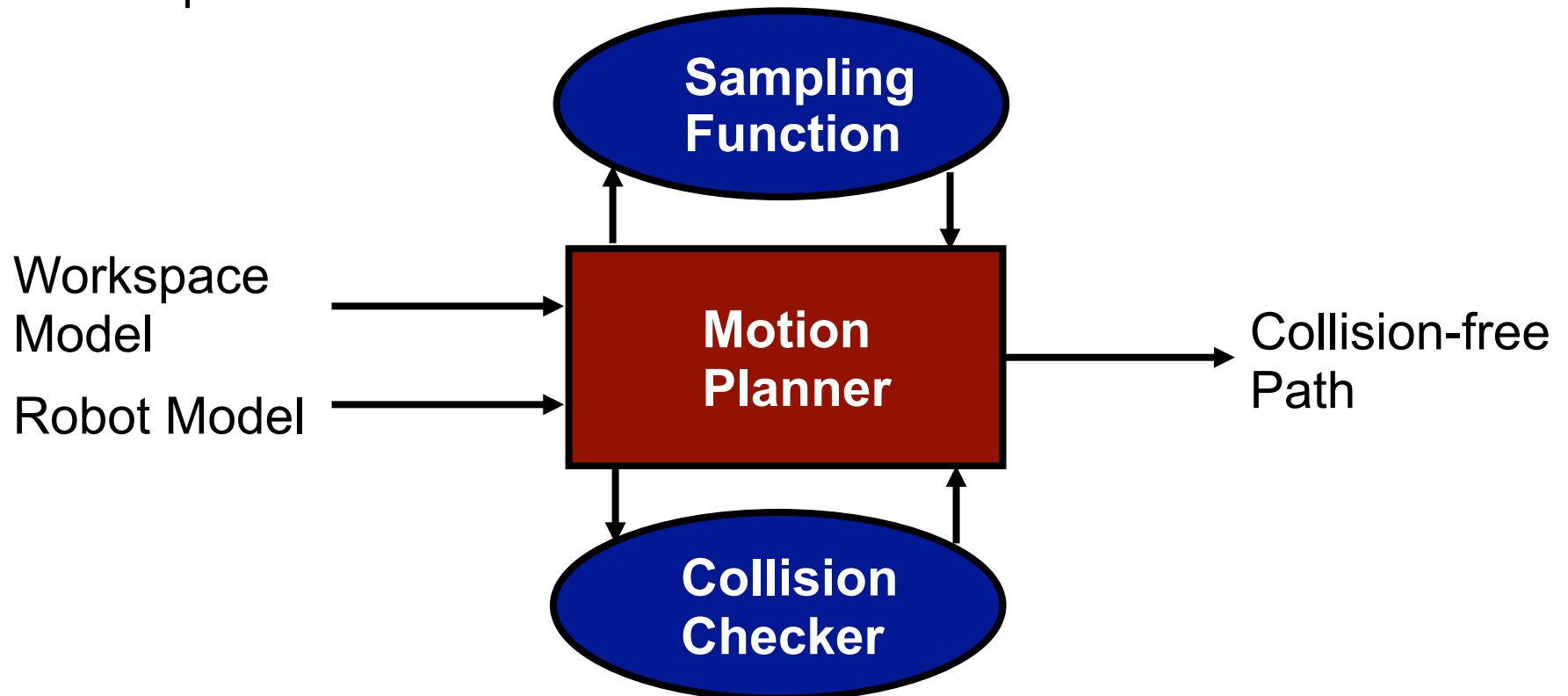
# Probabilistic Roadmap (PRM)





# Key components of the planner

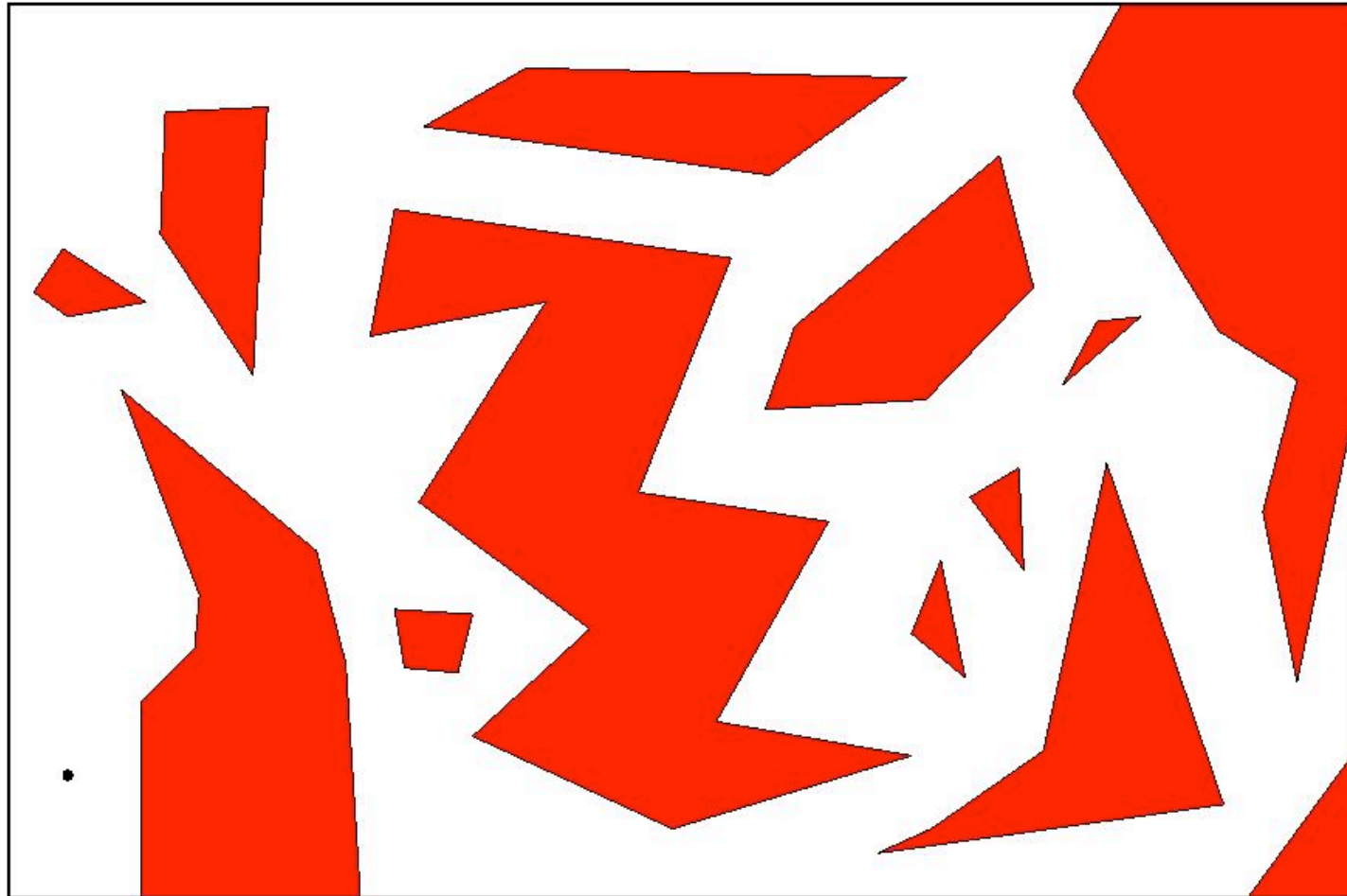
- Nearest neighbor datastructures
- Sampling strategy [AB98,HR03,OS99,BL01]
- Connection strategy
- Collision checking [LM97,LM03]
- Graph search







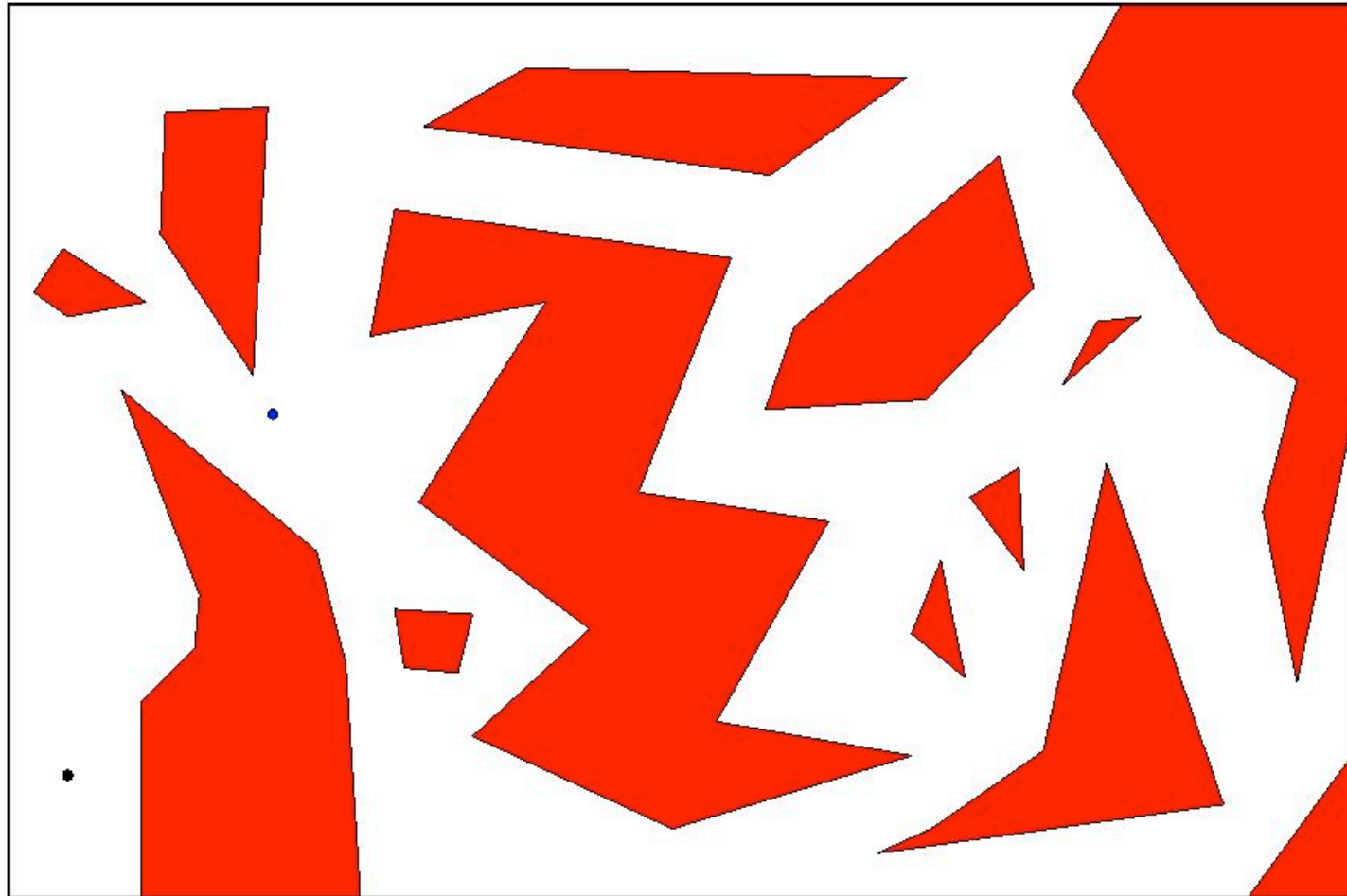
# Tree-based planner (RRT)





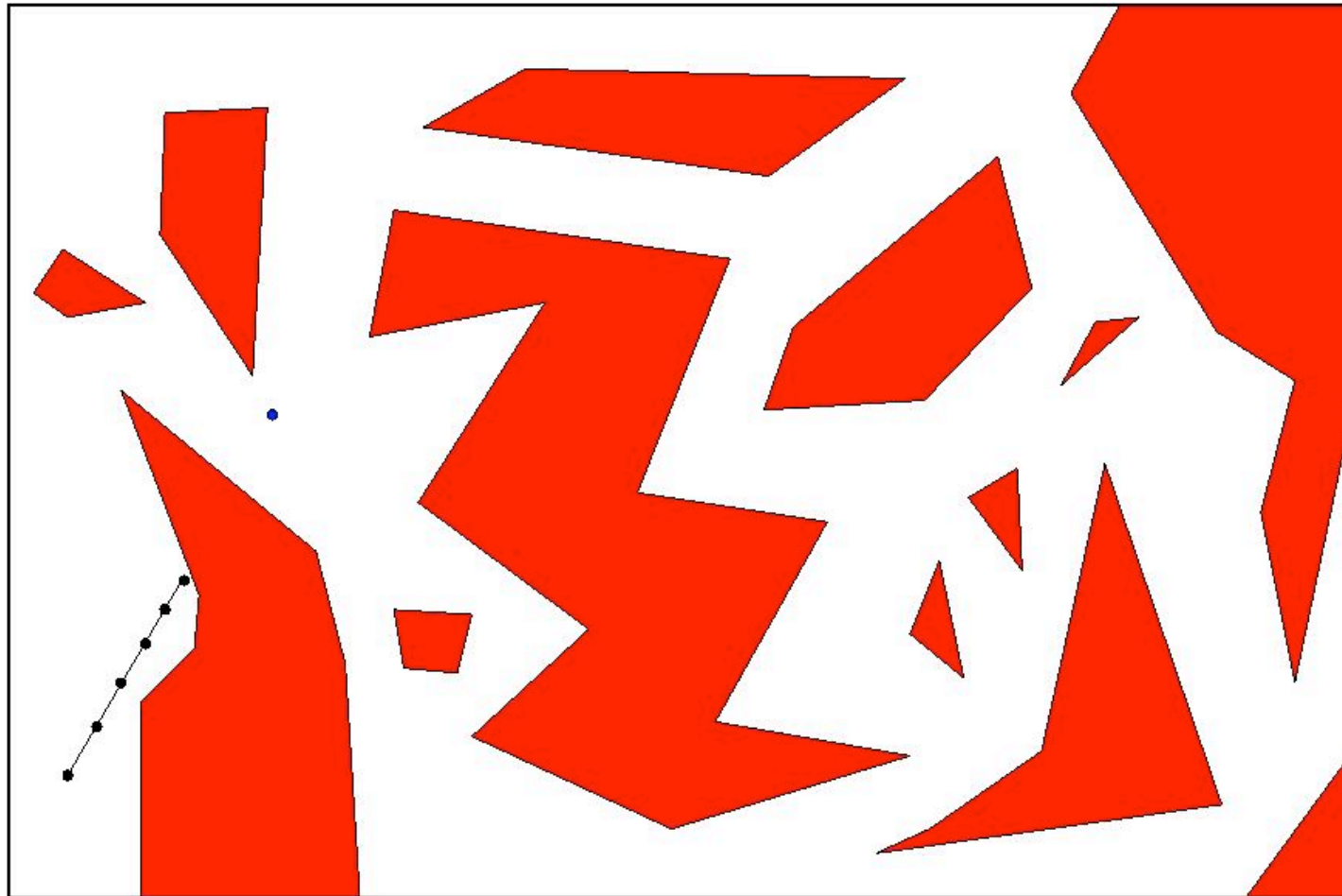


# Tree-based planner (RRT)



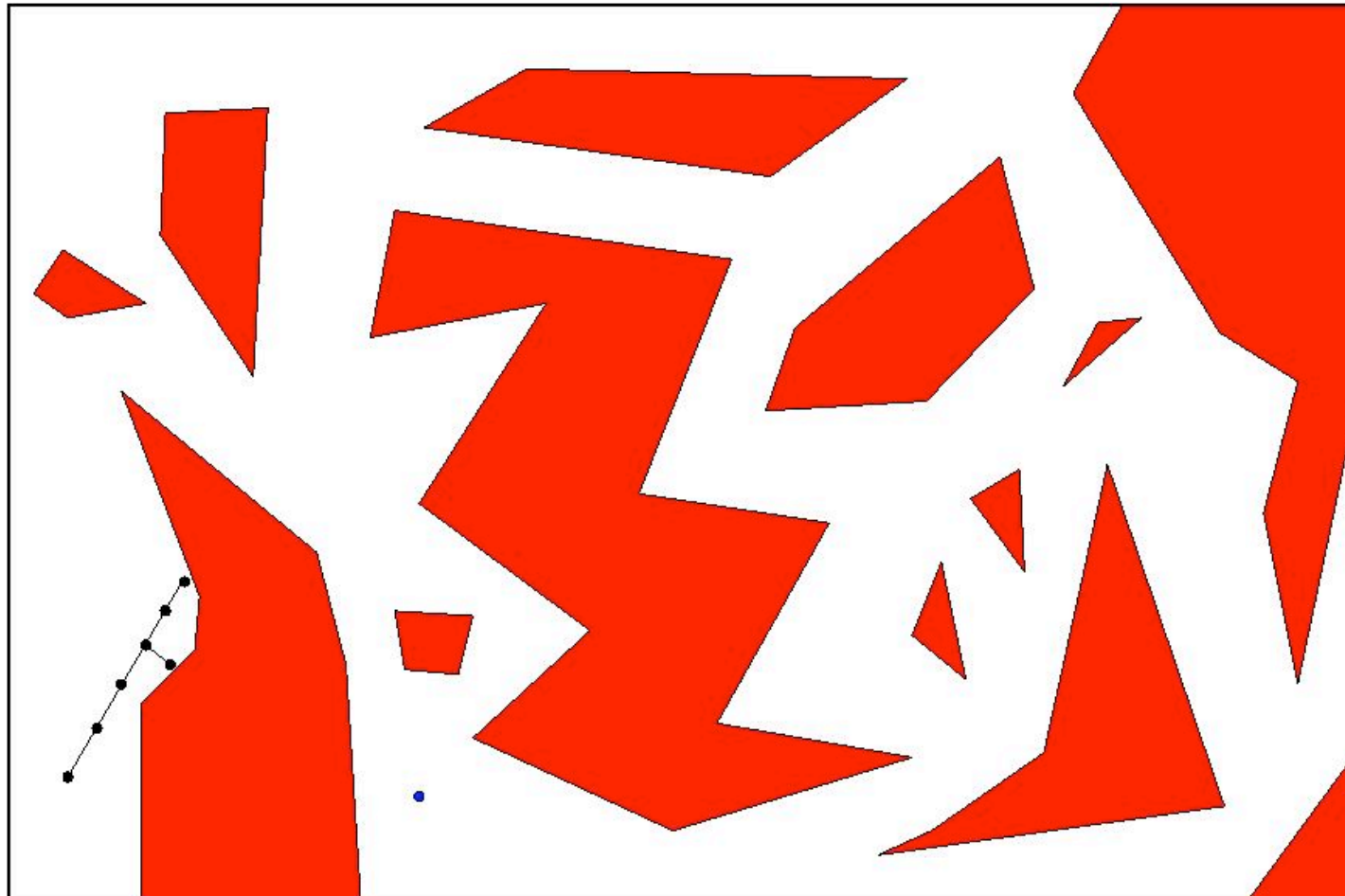


# Tree-based planner (RRT)



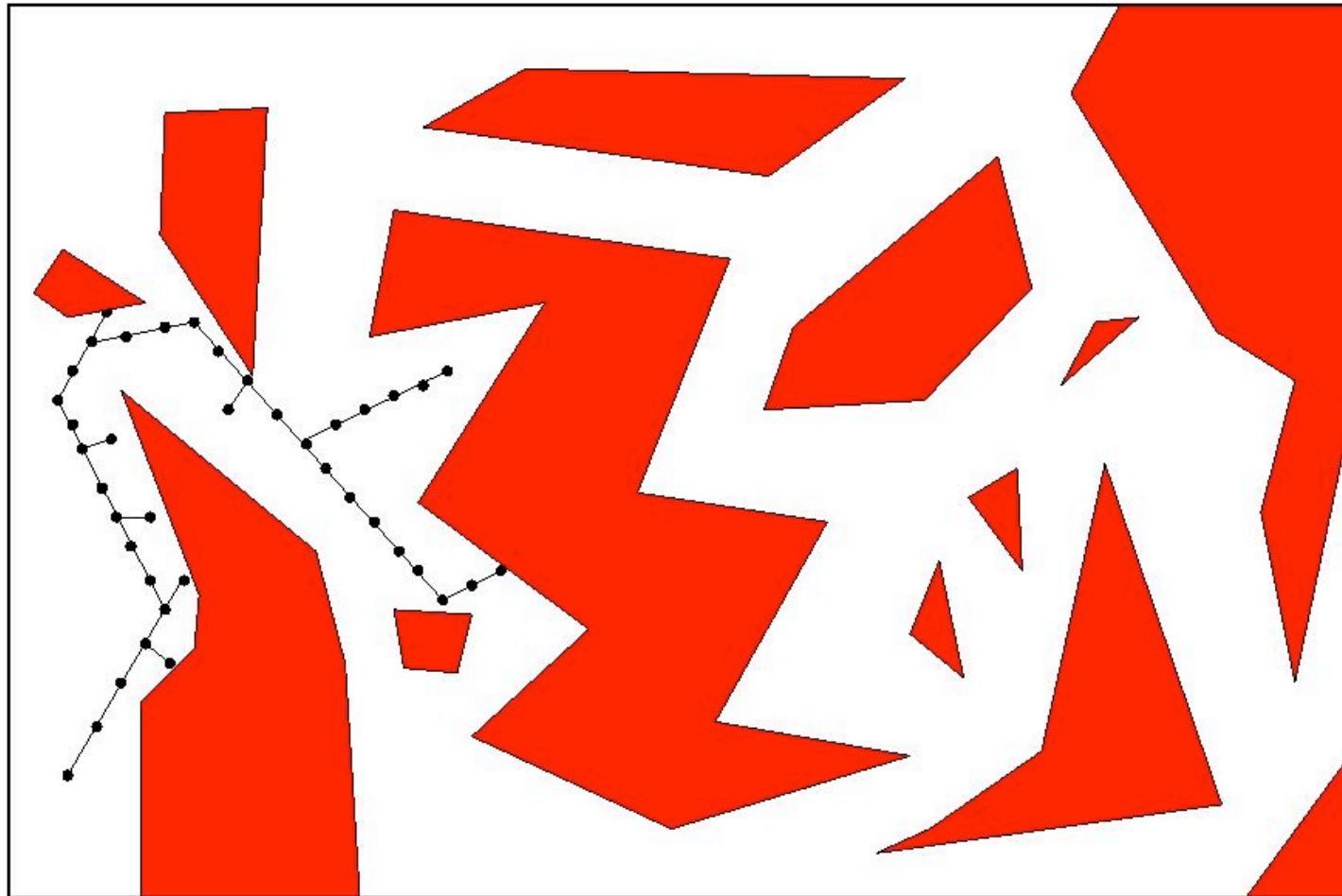


# Tree-based planner (RRT)





# Tree-based planner (RRT)





# Tree-based planners

- Rapidly-exploring Random Tree (RRT) [KL99,LK01]
- Expansive Space Tree (EST) [H97,H00]
- Single Query Bidirectional Lazy Tree (SBL) [SL01]
- Guided Expansive Space Tree [PK03]
- Adaptive Dynamic Domain RRT [Y05]
- Utility-guided RRT [BB07]
- Particle RRT [NR07]
- ...

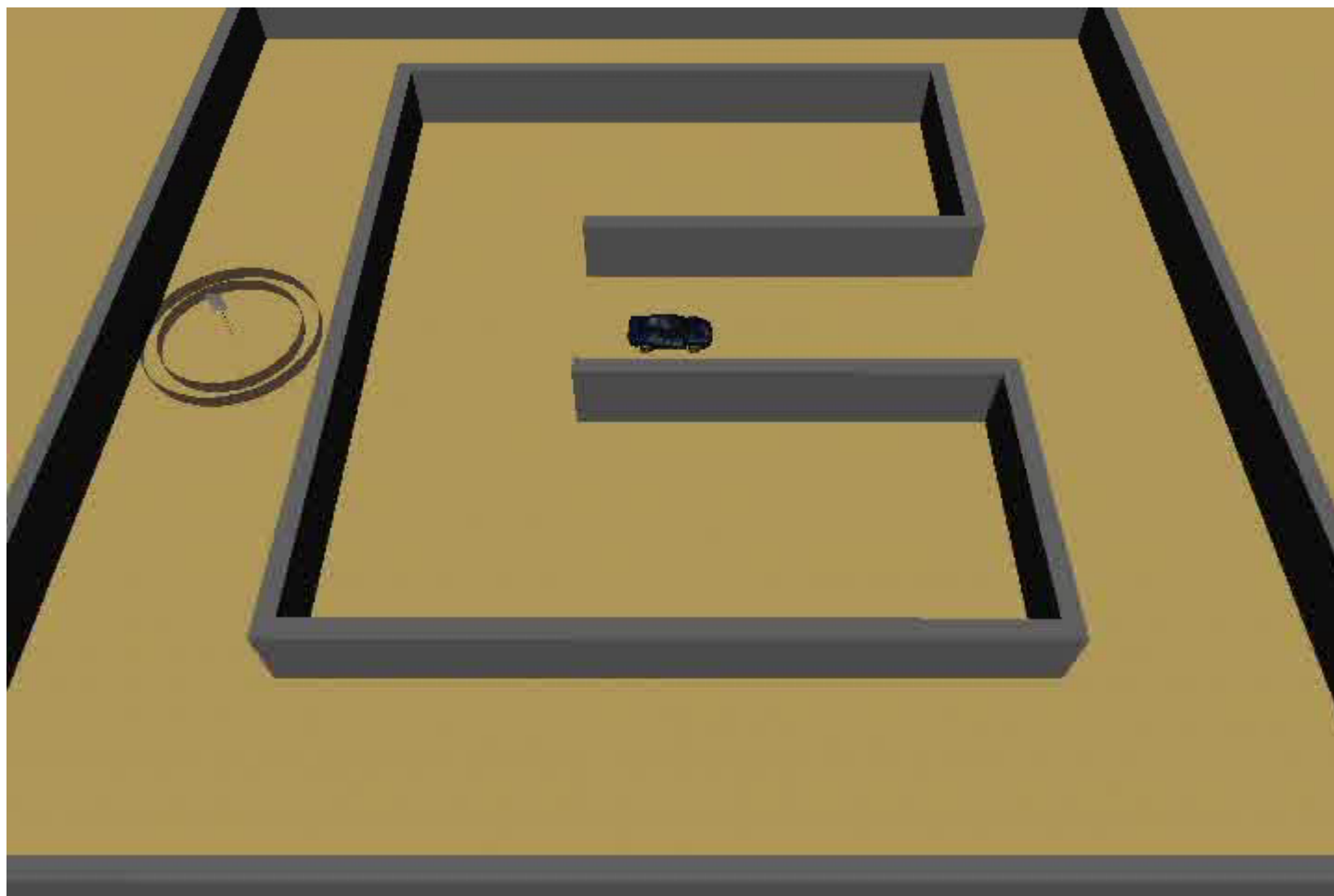


- Real robots have dynamic and actuation constraints
- Tree-based planners can easily accommodate a forward dynamics simulator to plan under realistic motion constraints
- Trees are directed data structures that nicely encode the notion of time
- Propagation is control based



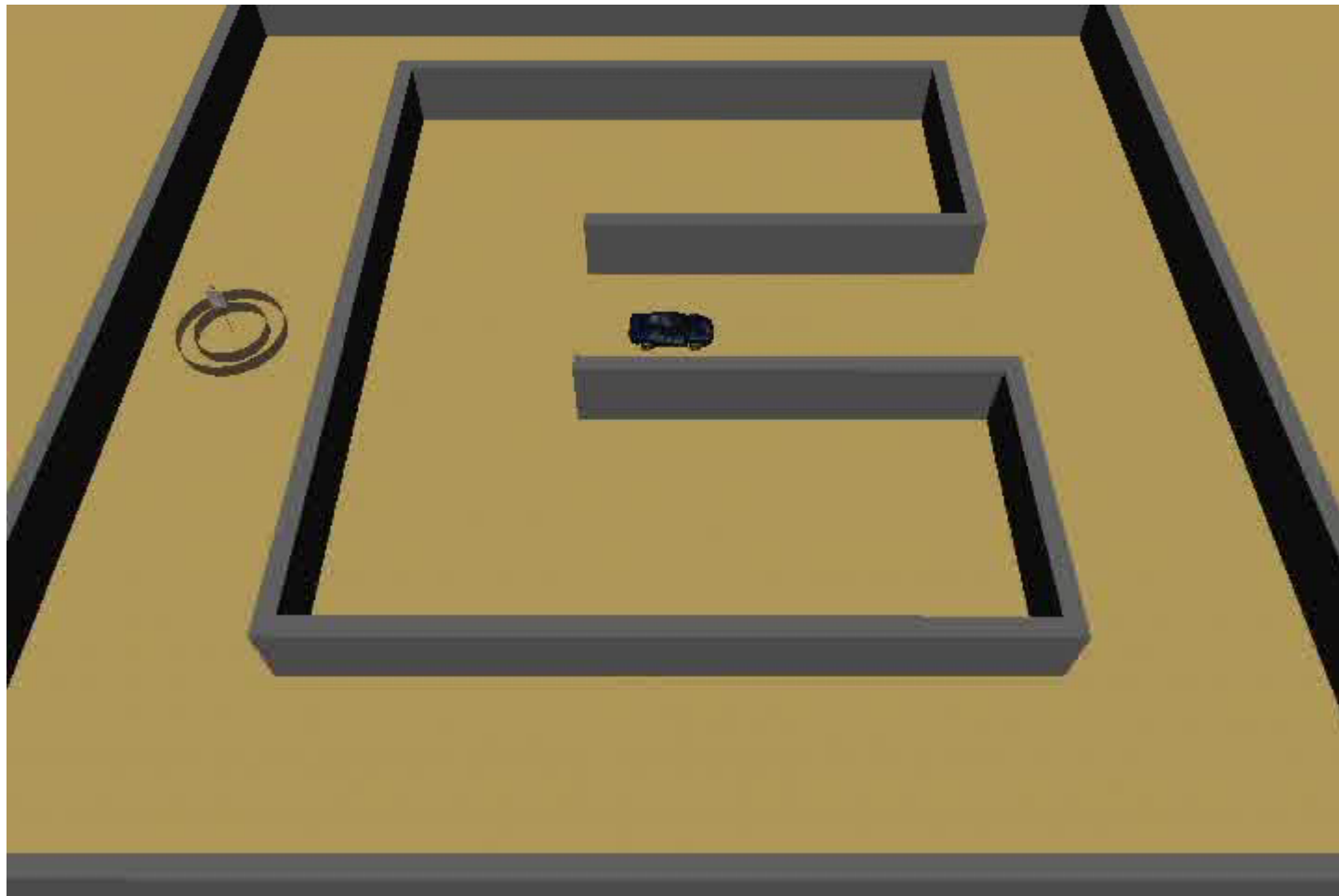


# Path planning for a car (PRM)



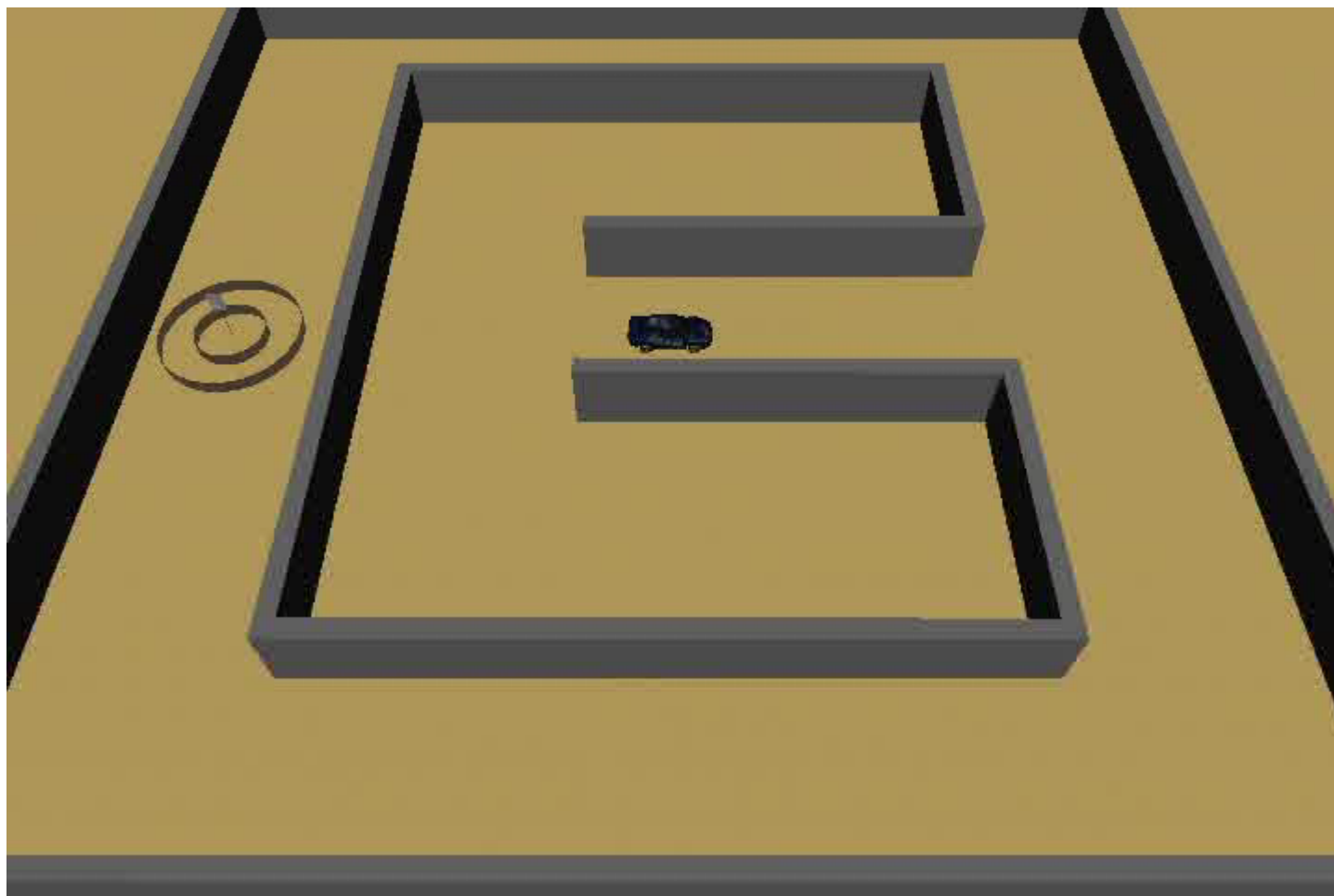


# Planning with velocity control (IST)



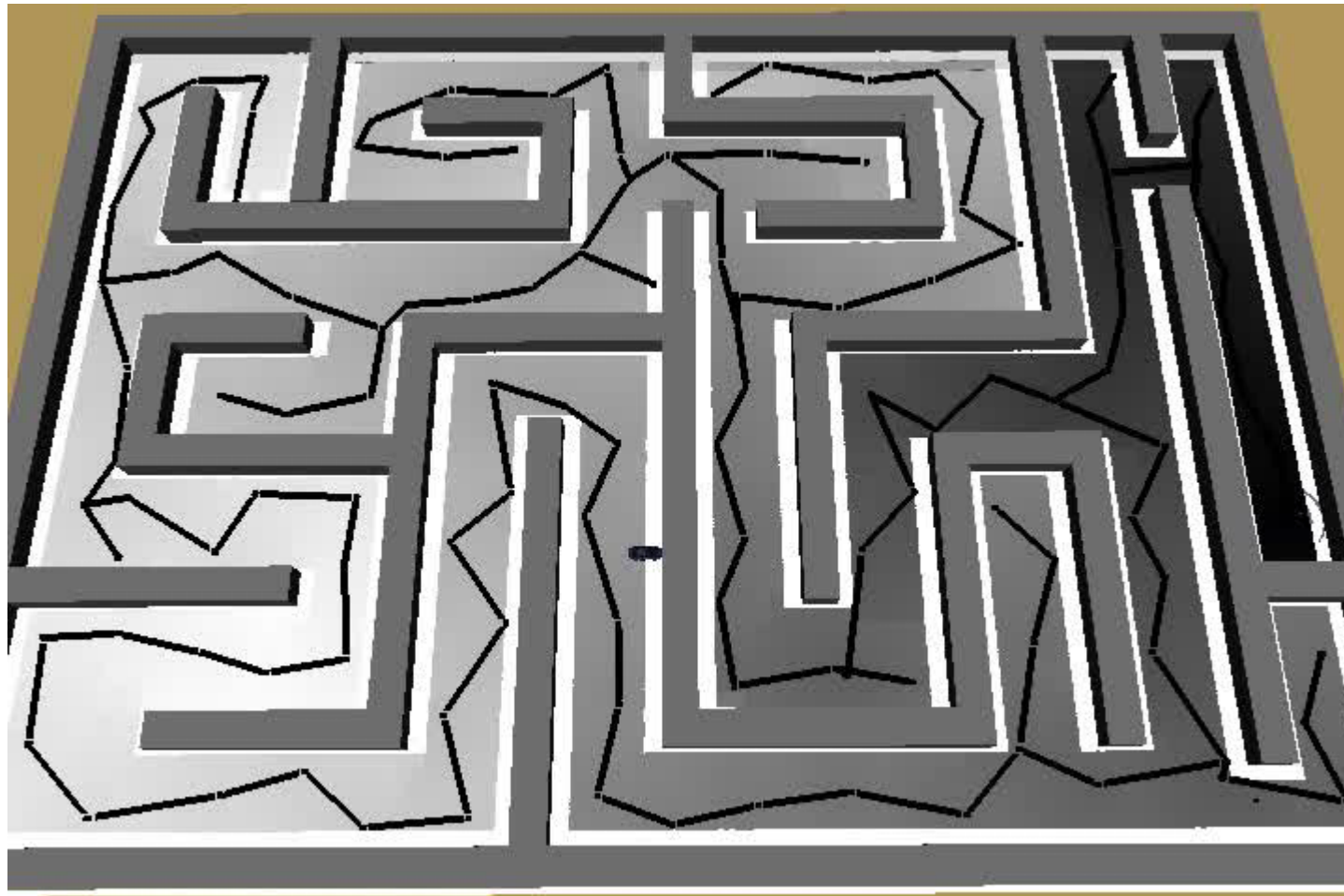


# Planning with acceleration control (IST)





# Building a tree with IST



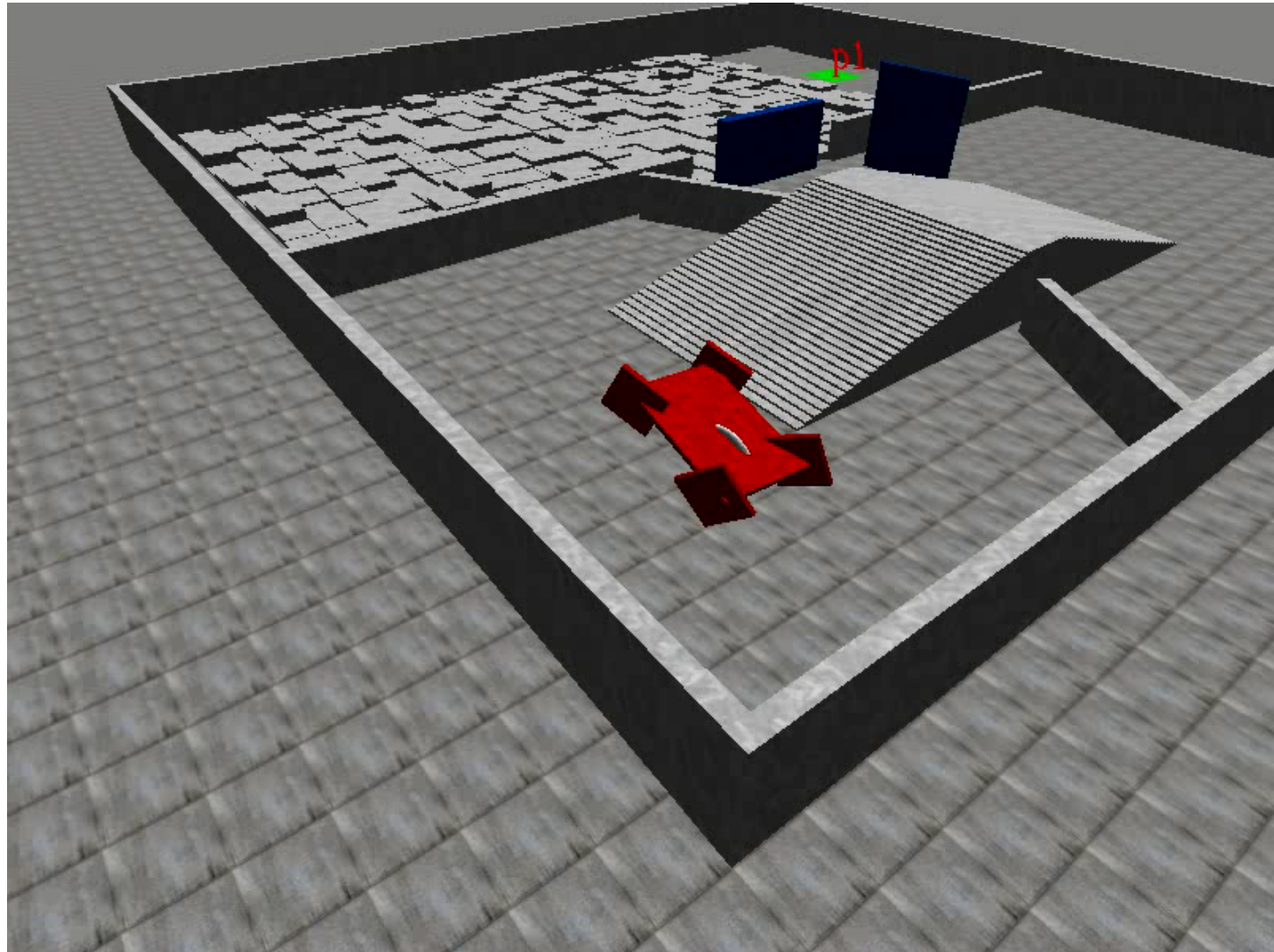


## More examples from the latest planners

- DSLX
- KPIECE
- Real-time replanning
- Real-time multirobot exploration



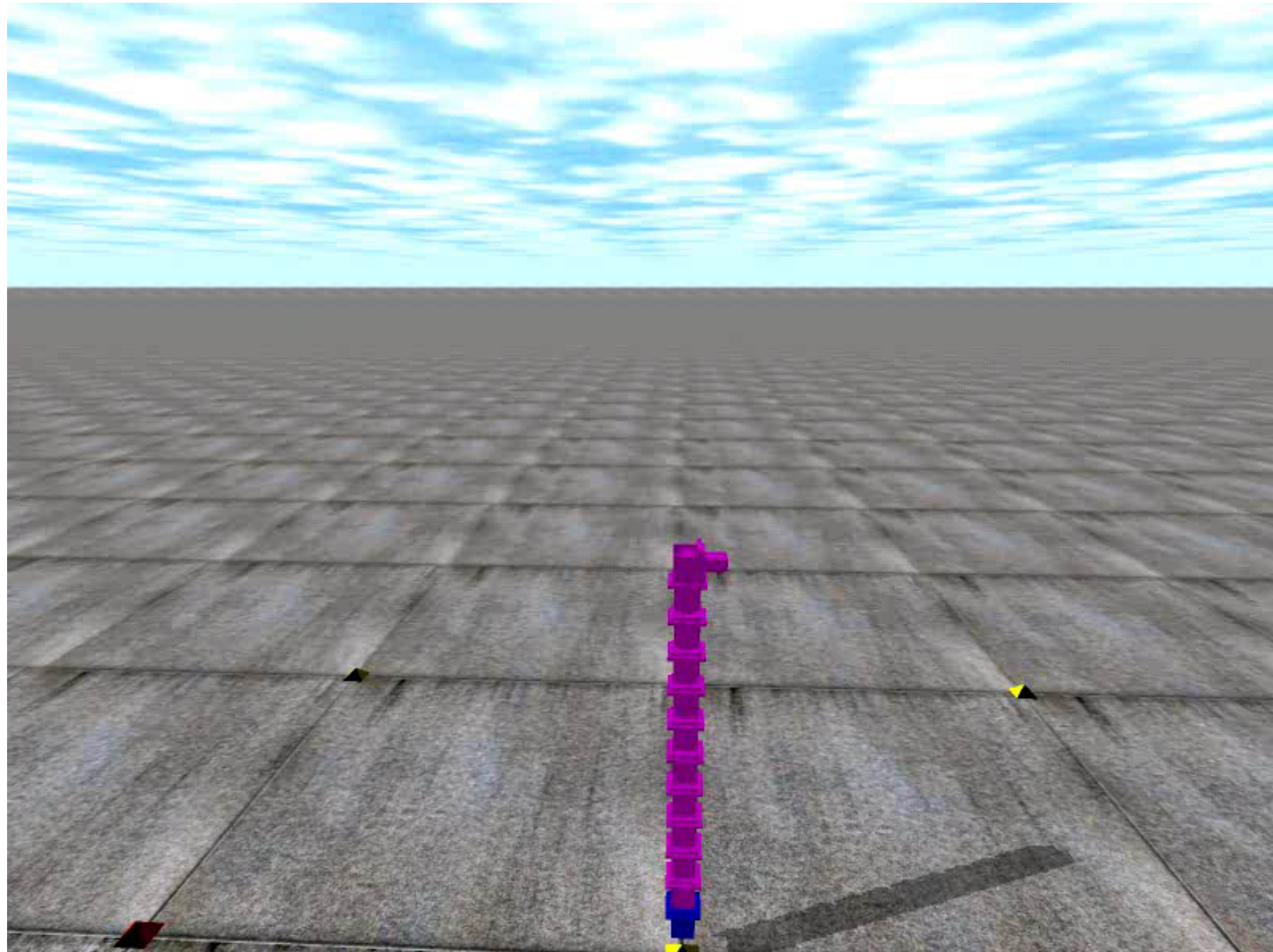
## Combining discrete search with continuous exploration







## Planning in high dimensional state and control spaces

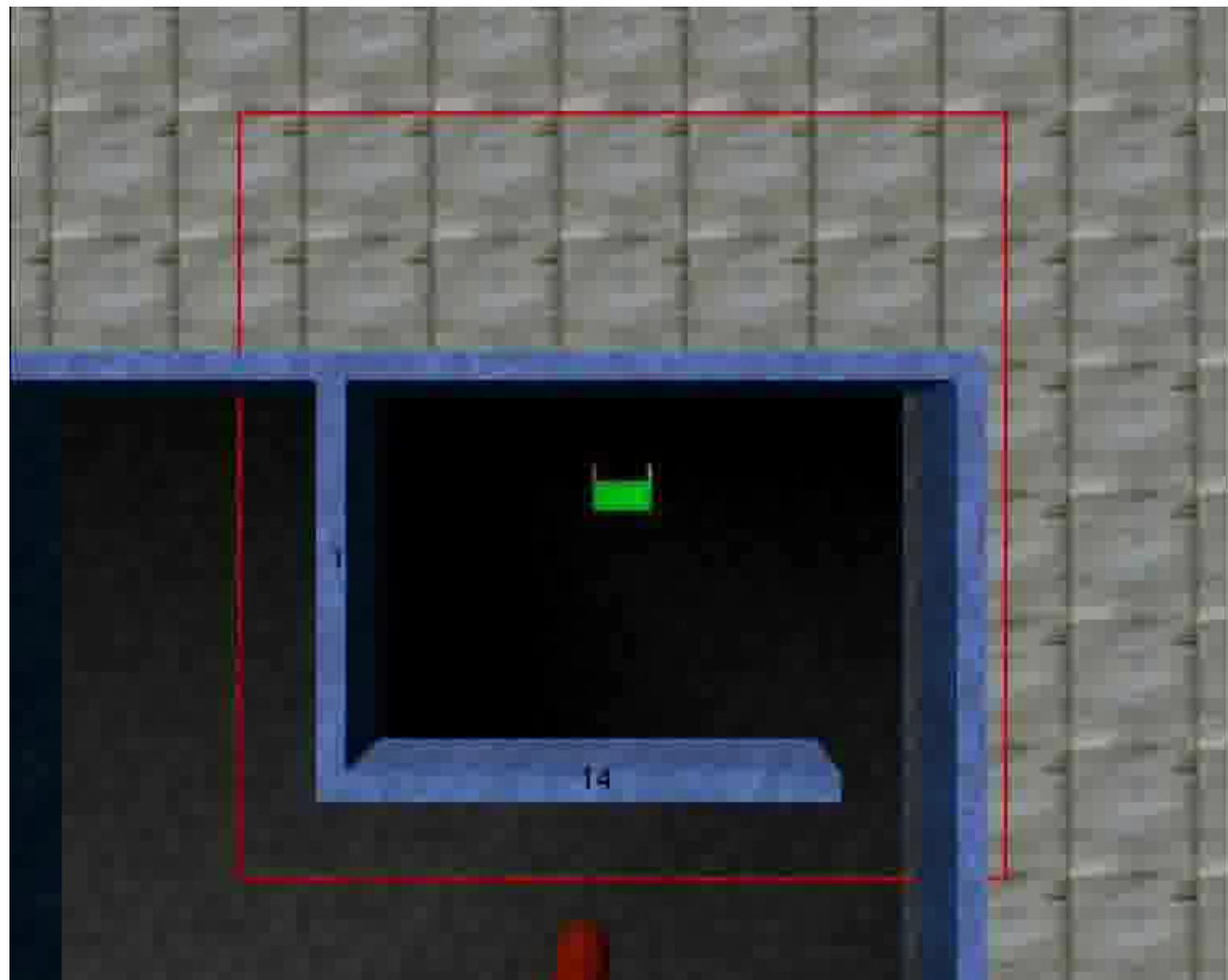






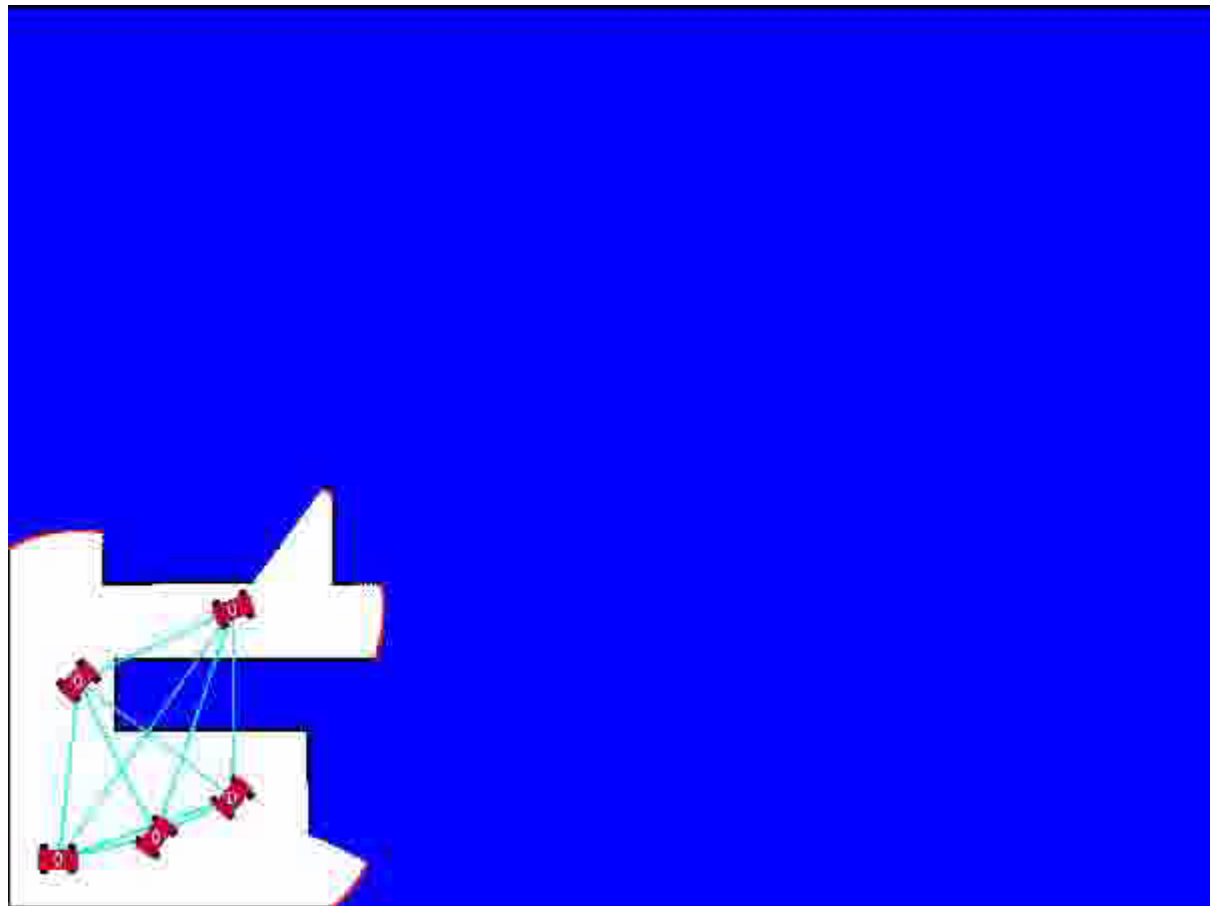
# Real-time replanning

Planning in partially known environments  
within moving obstacles





# Multirobot coordinated exploration





## Some facts about sampling-based planners

- General and applicable to many different systems
  - Very efficient on average, yielding solutions to many previously intractable problems
  - Relatively simple to describe and implement
  - Subject to problem specific optimizations
- 
- At best only probabilistically complete
  - Face difficulties in dealing with narrow passages



## And some more facts...

- Sampling-based planners contain many standardized building blocks
- As planners get more complicated it is useful to not have to code the basic components again and again
- There is not common benchmarking platform and planning problems
- Performance comparisons are not always fair
- Speedups may be misleading due to non-uniform implementation details



## And some more facts...

- Sampling-based planners contain many standardized building blocks
- As planners get more complicated it is useful to not have to code the basic components again and again
- **There is not common benchmarking platform and planning problems**
- Performance comparisons are not always fair
- Speedups may be misleading due to non-uniform implementation details



## Where OOPSMP comes in

- Different planners can be tested on the exact same planning scenario
- All planners use common data structures and utilities
- “Quick and dirty” testing is possible due to many already existing planners and modules
- The focus can be on algorithmic aspects rather than implementation details
- Can be integrated with different visualization software and physics simulators for nicer looking and more realistic results
- OOPSMP is currently unique at providing a large variety of sampling-based motion planners, data structures and building blocks that are tuned for motion planning applications



Thank you!